Worksheet 1.1 Fractions

Section 1 Simplifying Fractions

Fractions arise often in everyday life. We use them when shopping, when cooking and when building. Numeric fractions have the form

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}}$$

where the numerator and the denominator are usually whole numbers. Numeric fractions are also called rational numbers. Notice that

$$b = \frac{b}{1}$$

where b is any number. So all numbers can be expressed as fractions in this way. Indeed there is more than one way to represent any fraction. Fractions which represent the same quantity in different ways are called *equivalent fractions.*

For instance $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ are equivalent fractions since they are all different ways of writing one half.

Indeed

$$\frac{x}{y} = \frac{x \times n}{y \times n}$$

where x, y and n are any numbers. This is because

$$\begin{aligned} \frac{n}{n} &= 1 \\ \text{and} \quad \frac{x \times n}{y \times n} &= \frac{x}{y} \times \frac{n}{n} \\ &= \frac{x}{y} \times 1 \\ &= \frac{x}{y} \end{aligned}$$

We use equivalent fractions in the arithmetic of fractions. Fractions in their simplest form have the property that the numerator and denominator have no common factors. See worksheet 1.2 to find out about factors if you need to.

To simplify fractions we break the numerator into factors and the denominator into factors and then we cancel common factors to leave us with an equivalent fraction in its simplest form.

Example 1 :

$$\begin{aligned}\frac{12}{24} &= \frac{12\times 1}{12\times 2}\\ &= \frac{12}{12}\times\frac{1}{2}\\ &= 1\times\frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$

Example 2 :

$$\begin{aligned}\frac{36}{15} &= \frac{12\times 3}{5\times 3}\\ &= \frac{12}{5}\times\frac{3}{3}\\ &= \frac{12}{5}\end{aligned}$$

Example 3 :

$$\begin{aligned}\frac{30}{45} &= \frac{6\times 5}{9\times 5}\\ &= \frac{2\times \cancel{3}\times \cancel{5}}{3\times \cancel{3}\times \cancel{5}}\\ &= \frac{2}{3}\end{aligned}$$

Note: A line through common factors in the numerator and the denominator helps to keep a track of working and is called canceling.

Exercises:

1. Simplify the following fractions
   (a) $\frac{8}{18}$
   (b) $\frac{16}{24}$
   (c) $\frac{27}{57}$
   (d) $\frac{180}{244}$
   (e) $\frac{256}{3250}$

Section 2 Mixed Numbers and Improper Fractions

A mixed number is one which has both an integer part and a fractional part, for instance $2\frac{1}{2}$ is a mixed number. When these appear in calculations you need to convert them to an equivalent improper fraction to make the calculations easier to perform. An improper fraction is one in which the numerator is larger than the denominator. This means that the fraction is bigger than 1. To convert a mixed number to an improper fraction, take the whole number part and replace it with an equivalent fraction with denominator the same as the fractional part. Then add this fraction to the fractional part to get the equivalent improper fraction.

Example 1 :

$$\begin{aligned} 2\frac{1}{2} &= \frac{2}{1} + \frac{1}{2} \\ &= \frac{2}{1} \times \frac{2}{2} + \frac{1}{2} \\ &= \frac{4}{2} + \frac{1}{2} \\ &= \frac{4+1}{2} \\ &= \frac{5}{2} \end{aligned}$$

Example 2 :

$$\begin{aligned} 1 + \frac{6}{7} &= \frac{1}{1} + \frac{6}{7} \\ &= \frac{1}{1} \times \frac{7}{7} + \frac{6}{7} \\ &= \frac{7}{7} + \frac{6}{7} \\ &= \frac{13}{7} \end{aligned}$$

The improper fractions so obtained can then be treated as any other fraction in the calculations performed.

Example 3 : Changing an improper fraction to a mixed numeral.

$$\begin{aligned} \frac{18}{5} &= \frac{15}{5} + \frac{3}{5} \\ &= 3 + \frac{3}{5} \\ &= 3\frac{3}{5} \end{aligned}$$

Exercises:

1. Change the following mixed numerals to improper fractions
   (a) $1\frac{2}{3}$
   (b) $5\frac{3}{8}$
   (c) $2\frac{1}{4}$
   (d) $3\frac{2}{5}$
2. Change the following improper fractions to mixed numerals
   (a) $\frac{18}{7}$
   (b) $\frac{24}{9}$
   (c) $\frac{5}{3}$
   (d) $\frac{46}{5}$

Section 3 Multiplication and division of fractions

Multiplication is the simplest of fraction operations since

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

That is

$$\text{fraction} \times \text{fraction} = \frac{\text{product of numerators}}{\text{product of denominators}}$$

The only complication is remembering to cancel any common factors; this is easier if you simplify as much as possible before performing the multiplication.

Example 1 :

$$\begin{aligned} \frac{5}{9} \times \frac{6}{7} &= \frac{5 \times 6}{9 \times 7} \\ &= \frac{5 \times \not{3} \times 2}{3 \times \not{3} \times 7} \\ &= \frac{10}{21} \end{aligned}$$

Example 2 :

$$\begin{aligned}\frac{1}{2}\times\frac{4}{7} &= \frac{1\times 4}{2\times 7}\\ &= \frac{1\times 2\times \cancel{2}}{\cancel{2}\times 7}\\ &= \frac{2}{7}\end{aligned}$$

Example 3 :

$$\begin{aligned}\frac{13}{3}\times\frac{9}{26} &= \frac{13\times 9}{3\times 26}\\ &= \frac{\cancel{13}\times \cancel{3}\times 3}{\cancel{3}\times 2\times \cancel{13}}\\ &= \frac{3}{2}\end{aligned}$$

When multiplying mixed numbers, change the mixed numeral(s) to an improper fraction before multiplying.

Example 4 :

$$\begin{aligned}1\frac{2}{3}\times 2\frac{1}{4} &= \frac{5}{3}\times\frac{9}{4}\\ &= \frac{15}{4}\\ &= 3\frac{3}{4}\end{aligned}$$

Example 5 :

$$\begin{aligned}2\frac{3}{5}\times 1\frac{5}{13} &= \frac{13}{5}\times\frac{18}{13}\\ &= \frac{18}{5}\\ &= 3\frac{3}{5}\end{aligned}$$

We now look at division of fractions. The reciprocal of any rational number x is $\frac{1}{x}$. It is the thing you need to multiply x by to get 1. The reciprocal of a fraction $\frac{a}{b}$ is again the thing you need to multiply it by to get 1 and since

$$\frac{a}{b}\times\frac{b}{a}=\frac{a\times b}{b\times a}=1$$

the reciprocal of any fraction is given by its reversal. So the numerator becomes the denominator and the denominator becomes the numerator.

Example 6 : The reciprocal of $\frac{1}{2}$ is $\frac{2}{1} = 2$.

Example 7 : The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$.

Example 8 : The reciprocal of $3 = \frac{3}{1}$ is $\frac{1}{3}$.

Example 9 : The reciprocal of $2\frac{2}{3} = \frac{8}{3}$ is $\frac{3}{8}$. (That is, to find the reciprocal of a mixed number, we need to change it to a proper fraction).

Division of fractions makes use of the reciprocal. Dividing by a fraction is the same as multiplying by its reciprocal. So

$$\frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x}$$

To see why let's do an example.

$$\begin{aligned}
\frac{a}{b} \div \frac{x}{y} &= \frac{\frac{a}{b}}{\frac{x}{y}} \\
&= \frac{\frac{a}{b}}{\frac{x}{y}} \times \frac{\frac{y}{x}}{\frac{y}{x}} \\
&= \frac{\frac{a}{b} \times \frac{y}{x}}{\frac{x}{y} \times \frac{y}{x}} \\
&= \frac{\frac{a}{b} \times \frac{y}{x}}{1} \\
&= \frac{a}{b} \times \frac{y}{x}
\end{aligned}$$

Note: There is no need to do all this in normal working. You can skip from $\frac{a}{b} \div \frac{x}{y}$ to $\frac{a}{b} \times \frac{y}{x}$ straight away.

Example 10 :

$$\begin{aligned} \frac{5}{8} \div \frac{1}{4} &= \frac{5}{8} \times \frac{4}{1} \\ &= \frac{5 \times 4}{2 \times 4} \\ &= \frac{5}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

Example 11 :

$$\begin{aligned} \frac{(\frac{6}{7})}{(\frac{3}{7})} &= \frac{6}{7} \div \frac{3}{7} \\ &= \frac{6}{7} \times \frac{7}{3} \\ &= \frac{\cancel{3} \times 2 \times \cancel{7}}{\cancel{7} \times \cancel{3}} \\ &= 2 \end{aligned}$$

Example 12 :

$$\begin{aligned} 2\frac{1}{2} \div 1\frac{3}{5} &= \frac{5}{2} \div \frac{8}{5} \\ &= \frac{5}{2} \times \frac{5}{8} \\ &= \frac{25}{16} \\ &= 1\frac{9}{16} \end{aligned}$$

Exercises:

1. Perform the following multiplications

   (a) $\frac{3}{5} \times \frac{7}{9}$

   (b) $\frac{13}{14} \times \frac{6}{10}$

   (c) $1\frac{2}{5} \times 2\frac{4}{7}$

   (d) $4\frac{1}{2} \times 1\frac{5}{21}$

2. Find the reciprocal of these numbers

(a) $\frac{2}{3}$

(b) $1\frac{1}{2}$

(c) $5\frac{2}{7}$

(d) $\frac{1}{8}$

3. Perform the following divisions

(a) $\frac{3}{5} \div \frac{6}{7}$

(b) $\frac{6}{11} \div \frac{9}{20}$

(c) $3\frac{1}{4} \div 1\frac{1}{2}$

(d) $8\frac{7}{12} \div 3\frac{3}{4}$

Section 4 Addition and Subtraction of Fractions

The addition and subtraction of fractions is slightly more complicated than the multiplication and division of them. The reason for this is that to add or subtract fractions, both fractions must have the same denominator. Given two fractions with the same denominator we do the addition and subtraction of the numerators and leave the denominator unchanged. If, for instance, we were adding three quarters and two quarters, the answer would be five quarters. To write this in a maths sentence we write

$$\frac{3}{4} + \frac{2}{4} = \frac{3+2}{4}$$

More generally, we would write

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \text{and} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Once you have the same denominator you make one fraction with the operation applying only to the numerator.

Example 1 : $\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

Example 2 : $\frac{5}{12} + \frac{2}{12} = \frac{5+2}{12} = \frac{7}{12}$

Example 3 : $\frac{6}{7} - \frac{1}{7} = \frac{6-1}{7} = \frac{5}{7}$

So what can we do when we are asked to calculate $\frac{1}{3} + \frac{1}{2}$? We need to take each fraction in the expression and replace it with an equivalent fraction, with the denominator of each equivalent fraction the same. To find which equivalent fractions are required we first find the lowest common multiple of the denominators. See worksheet 1.2 if you are unsure about lowest common denominators.

So for $\frac{1}{3} + \frac{1}{2}$ we look for the lowest common multiple of 2 and 3. This is 6. We now replace $\frac{1}{3}$ with an equivalent fraction which has denominator 6. We repeat this process for $\frac{1}{2}$.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \qquad \text{and} \qquad \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

Therefore,

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{2+3}{6} = \frac{5}{6}$$

Example 4 :

$$\begin{aligned} \frac{1}{3} + \frac{1}{4} &= \frac{1}{3} \times \frac{4}{4} + \frac{1}{4} \times \frac{3}{3} \\ &= \frac{4}{12} + \frac{3}{12} \\ &= \frac{4+3}{12} \\ &= \frac{7}{12} \end{aligned}$$

Click here and then here to see this example done for you in a video.

Example 5 :

$$\begin{aligned} \frac{1}{5} + \frac{3}{10} &= \frac{1}{5} \times \frac{2}{2} + \frac{3}{10} \\ &= \frac{2}{10} + \frac{3}{10} \\ &= \frac{2+3}{10} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$$

Example 6 :

$$\begin{aligned} \frac{5}{8} - \frac{5}{12} &= \frac{5}{8} \times \frac{3}{3} - \frac{5}{12} \times \frac{2}{2} \\ &= \frac{15}{24} - \frac{10}{24} \\ &= \frac{15 - 10}{24} \\ &= \frac{5}{24} \end{aligned}$$

Example 7 :

$$\begin{aligned} 1\frac{2}{3} + 3\frac{1}{4} &= 1 + \frac{2}{3} + 3 + \frac{1}{4} \\ &= 4 + \frac{2}{3} \times \frac{4}{4} + \frac{1}{4} \times \frac{3}{3} \\ &= 4 + \frac{8}{12} + \frac{3}{12} \\ &= 4\frac{11}{12} \end{aligned}$$

Example 8 :

$$\begin{aligned} 2\frac{4}{5} + 1\frac{5}{8} &= 2 + \frac{4}{5} + 1 + \frac{5}{8} \\ &= 3 + \frac{4}{5} \times \frac{8}{8} + \frac{5}{8} \times \frac{5}{5} \\ &= 3 + \frac{32}{40} + \frac{25}{40} \\ &= 3 + \frac{57}{40} \\ &= 3 + 1\frac{17}{40} \\ &= 4\frac{17}{40} \end{aligned}$$

Example 9 :

$$\begin{aligned}6\frac{5}{8} - 4\frac{1}{4} &= 6 + \frac{5}{8} - \left(4 + \frac{1}{4}\right)\\ &= 6 + \frac{5}{8} - 4 - \frac{1}{4} \times \frac{2}{2}\\ &= 6 - 4 + \frac{5}{8} - \frac{2}{8}\\ &= 2 + \frac{3}{8}\\ &= 2\frac{3}{8}\end{aligned}$$

Example 10 :

$$\begin{aligned}4\frac{1}{7} - 1\frac{3}{5} &= 4 + \frac{1}{7} - \left(1 + \frac{3}{5}\right)\\ &= 4 + \frac{1}{7} \times \frac{5}{5} - 1 - \frac{3}{5} \times \frac{7}{7}\\ &= 4 + \frac{5}{35} - 1 - \frac{21}{35}\\ &= 3 + \frac{5}{35} - \frac{21}{35}\\ &= 2 + 1 + \frac{5}{35} - \frac{21}{35}\\ &= 2 + \frac{35}{35} + \frac{5}{35} - \frac{21}{35}\\ &= 2 + \frac{40}{35} - \frac{21}{35}\\ &= 2\frac{19}{35}\end{aligned}$$

Exercises:

1. Evaluate the following additions and subtractions
   (a) $\frac{2}{3} + \frac{4}{7}$
   (b) $\frac{5}{8} - \frac{1}{3}$
   (c) $1\frac{1}{2} + 2\frac{4}{5}$
   (d) $8\frac{1}{4} + 2\frac{7}{9}$
   (e) $9\frac{5}{8} - 2\frac{1}{4}$

(f) $7\frac{2}{3} - 1\frac{8}{9}$

(g) $5\frac{1}{4} - 1\frac{1}{2}$

Exercises for Worksheet 1.1

1. (a) Write these fractions in their simplest form.

   i. $\frac{4}{10}$ ii. $\frac{21}{35}$ iii. $\frac{45}{54}$

   (b) Find the lowest common denominator for each pair of fractions.

   i. $\frac{1}{2}, \frac{2}{5}$ ii. $\frac{1}{3}, \frac{3}{5}$ iii. $\frac{1}{3}, \frac{3}{4}, \frac{3}{9}$

   (c) Change these mixed numbers into improper fractions

   i. $2\frac{3}{7}$ ii. $6\frac{3}{8}$ iii. $4\frac{5}{12}$

   (d) Change these improper fractions to mixed numbers

   i. $\frac{56}{9}$ ii. $\frac{27}{6}$ iii. $\frac{37}{4}$

   (e) Find the reciprical of

   i. $\frac{1}{3}$ ii. 5 iii. $\frac{4}{9}$

   (f) Put each group of numbers in ascending size

   i. $\frac{1}{5}, \frac{7}{5}, \frac{3}{5}$
   ii. $\frac{1}{10}, \frac{1}{100}, \frac{1}{5}, \frac{1}{20}$
   iii. $\frac{3}{8}, \frac{2}{3}, \frac{5}{12}$

2. Evaluate the following

   (a) $\frac{1}{8} + \frac{4}{8}$
   (b) $\frac{5}{6} + \frac{3}{4}$
   (c) $\frac{16}{25} - \frac{3}{5}$
   (d) $\frac{1}{2} - \frac{1}{3} + \frac{4}{9}$
   (e) $2\frac{1}{2} + 3\frac{1}{4}$
   (f) $\frac{3}{8} \times \frac{5}{7}$
   (g) $\frac{6}{7} \times \frac{14}{15}$
   (h) $\frac{1}{3} \div \frac{9}{5}$
   (i) $\frac{1}{100} \div \frac{1}{10}$
   (j) $10\frac{1}{2} \div 5\frac{1}{4}$
   (k) $3\frac{1}{3} \times 2\frac{3}{10} \times 12$
   (l) $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

3. (a) What fraction of one kilometre is one centimetre?

   (b) What fraction of one day is one second?

   (c) There are 16 girls in a class of 40. What fraction of the class is made up of boys?

   (d) Two family size pizzas (one ham and pineapple, and the other supreme) are cut into 10 and 12 pieces respectively. Bill eats 3 pieces of the ham and pineapple and Rowan eats 4 pieces of the supreme. Has Bill or Rowan eaten more pizza?